Structural Tailoring and Feedback Control Synthesis: An Interdisciplinary Approach

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Structural tailoring provides an attractive method to optimize the performance of actively controlled space structures. However. the simultaneous optimization of control gains and structural properties often becomes prohibitively expensive for large systems, and physical insight is often lost in the resulting control law. This paper presents a method for optimization of the closed-loop structural system using only structural tailoring. Optimal linear quadratic regulator (LQR) control theory is used with weighting matrices chosen based on physical considerations. The LQR control law depends only on two scalar gains and the structural properties. Hence, the closed-loop performance can be expressed in terms of the structural parameters. Results are given for a beam and a truss beam to show the simplicity of the method and the importance of structural tailoring to increase dynamic performance and to reduce control effort.

Nomenclature

$\mathbf{A}$ = state matrix
a_j = real part of the jth circular frequency
$\mathbf{B}$ = control influence matrix
b_j = imaginary part of the jth circular frequency
$\mathbf{C}$ = damping matrix
$\mathbf{D}$ = actuator influence matrix
$\mathbf{E}$ = sensor influence matrix
E = Young's modulus
f = disturbance force vector
$\mathbf{G}$ = equal to one-half of the critical damping matrix
G = shear modulus
$\mathbf{I}$ = identity matrix
i = equal to $\sqrt{-1}$
J = scalar performance/cost index
$\mathbf{K}$ = stiffness matrix
ℓ = beam length
$\mathbf{M}$ = mass matrix
$\mathbf{P}$ = Riccati matrix for optimal control
$\mathbf{Q}$ = state weighting matrix
q = structural displacement vector
$\dot{q}$ = structural velocity vector
$\mathbf{R}$ = control force weighting matrix
$\mathbf{T}$ = eigenvector matrix
t = beam thickness
$\mathbf{U}$ = triple matrix product, Eq. (20)
u = control force vector
$\mathbf{V}$ = proportional control gain matrix
$\mathbf{W}$ = rate control gain matrix
x = state vector
$\dot{x}$ = temporal derivative of state vector
y = system output
z = modal displacement vector
$\dot{z}$ = modal velocity vector
α = scalar weighting of strain energy
β = scalar weighting of kinetic energy
γ = modal force participation coefficient
ω_j = jth circular frequency
Λ = diagonal eigenvalue matrix
η = scalar proportional control gain
ζ = scalar rate control gain
ρ = mass density

I. Introduction

APPLICATIONS of optimal designs in aerospace structural engineering are becoming increasingly important due to the high cost of spacecraft, more stringent performance requirements, and increasing emphasis on structural safety for space deployment. These considerations are particularly important in the design of space structures that employ active feedback control. A number of recent studies have considered the simultaneous optimization of both structure and control system, as can be found in Refs. 1–10. These works have demonstrated that tailoring of the structural mass and stiffness can improve performance and reduce the cost of active control. However, conventional control/structure optimization usually involves a large number of control gains which become part of the overall system design variables. For large problems, the number of control and structural design variables can make simultaneous optimization prohibitively expensive.

A recent approach to simultaneous control structure optimization, found for example in Refs. 8–10, has involved the use of optimal control theory with weighting matrices chosen to be functions of the structural mass and stiffness matrices. This method yields a control law that depends only on structural parameters and several scalar gains. Although such an approach does not guarantee the "best" control law, it does provide the advantage of reducing the number of design variables as compared to the conventional control/structure optimization procedure. Unfortunately, even when the control law is expressed in terms of the structural parameters, simultaneous optimization of controls and structures often reduces to a set of nonlinear constrained equations, which are difficult to interpret in terms of the underlying physics.

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For successful interdisciplinary control structure optimization, physical insight into the effect of structural tailoring on the closed-loop system is needed. To this end, we propose using optimal control theory with the weighting matrices of Refs. 8–10 in conjunction with modal-space control.[11] A closed-form solution for the control law that yields substantial physical insight into the proper structural tailoring objective is presented. The paper shows that weighting matrices that minimize system energy in the performance cost functional result in control gains that exploit the intrinsic characteristics of the structural stiffness and mass matrices. Hence, optimization of the closed-loop system can be performed using only structural tailoring. A structural tailoring objective is derived that is simple enough to be used in the initial design of actively controlled structures. Results for a simple beam and a more complicated truss structure are used to show the simplicity of the proposed structural tailoring procedure as well as the importance of tailoring structures to increase system performance and to decrease control effort.

II. Physical Choices for Optimal Control Weighting Matrices

Optimal linear quadratic (LQ) design techniques are frequently used to synthesize control laws for multi-input, multi-output systems. The synthesized control law, $\boldsymbol{u}$, is nonlinearly related to matrices that weight the cost of performance and control. The following paragraphs describe a choice of weighting matrices based on physical considerations that were also used in Refs. 8–10. First, the governing equations are described, then, the energy of the closed-loop system is used to choose the weighting matrices.

A. Governing Equations for Linear Quadratic Regulator Design

For the present discussion, we consider an unforced structural system with full state feedback control. The objective of the control law, known as a regulator, is to suppress vibrations which may have been induced by rigid body slewing or external disturbances. The equations of motion for this system may be written in discrete form as

$$\mathbf{M}\ddot{q} + \mathbf{C}\dot{q} + \mathbf{K}q = \mathbf{D}u, \qquad q(0) = q_0, \qquad \dot{q}(0) = \dot{q}_0 \tag{1}$$

Equation (1) may also be written in first-order, state variable form as

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \qquad y = \mathbf{E}x \tag{2}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

For the regulator design, we wish to minimize the response of the closed-loop system and the control effort simultaneously. This may be expressed using the following quadratic cost functional

$$J = \int_{t_0}^{t_1} \frac{1}{2} (x^T\mathbf{Q}x + u^T\mathbf{R}u)\, \mathrm{d}\tau \tag{3}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices which are based on the relative importance of each state and control force, respectively. The only restrictions on $\mathbf{Q}$ and $\mathbf{R}$ are that $\mathbf{Q}$ must be at least semidefinite, and $\mathbf{R}$ must be positive definite.

To minimize the preceding functional, the optimal control force can be derived as given in basic texts, such as in Ref. 12, to be

$$u = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}x \tag{4}$$

where $\mathbf{P}$ is the only positive definite solution to the steady-state matrix Riccati equation

$$0 = \mathbf{Q} + \mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} \tag{5}$$

Thus, it is obvious that the control gain $(-\mathbf{RB}^T\mathbf{P})$ is nonlinearly related to the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. Finding the "best" choice of $\mathbf{Q}$ and $\mathbf{R}$ can be one of the more difficult steps in the feedback control design process. Simultaneous optimization of structural properties as well as elements of the $\mathbf{Q}$ and $\mathbf{R}$ matrices becomes prohibitively expensive for large problems. In addition, it frequently leads to a total loss of physical insight into the resulting control gains. However, by choosing the weighting matrices a priori to yield consistent physical quantities in the cost index of Eq. (3) (e.g., energy, forces, displacements), one can arrive at a control law that preserves physical insight.

B. Minimization of Energy

The objective of many optimization problems involves minimizing the system energy. For structure/control problems, if the stiffness matrix is positive definite (i.e., there are no rigid body modes), a logical choice of weighting matrices to minimize the energy of the structure and the quasistatic "work" of the controller is

$$\mathbf{Q} = \begin{bmatrix} \alpha\mathbf{K} & 0 \\ 0 & \beta\mathbf{M} \end{bmatrix} \qquad \mathbf{R} = [\mathbf{D}^T\mathbf{K}^{-1}\mathbf{D}] \tag{6}$$

The coefficients α and β weight the structure's strain and kinetic energy. The work of the controller is implicitly weighted by α and β [$\alpha, \beta \geq 0$ and $(\alpha + \beta) > 0$]. Minimization of the controller work is relevant where both limited force and stroke are available as with inertial mass actuators. For cases where the stiffness matrix is not positive definite, rigid body motion could be removed by a reduction technique and the controller designed for flexible modes of vibration only. Equation (6) differs slightly from that given in Refs. 8–10 in that only two scalar weights are used.

Solution of the matrix Riccati equation with the weighting matrices of Eq. (6) yields a control law which depends only on the scalar weights α and β and the structural mass and stiffness matrices. As will be shown in the next section, a closed-form solution for the optimal control law is available when the actuator influence matrix, $\mathbf{D}$, is full rank.

III. Closed-Form Solution of the Riccati Equation

A closed-form solution of the Riccati equation can be easily derived for the preceding weighting matrices when the number of actuators is equal to the number of model degrees of freedom ($\mathbf{D}$ is full rank). Meirovitch[11] has exploited the case of $\mathbf{D}$ being full rank to develop modal-space control when the number of actuators is equal to the number of controlled modes. This paper reviews the case where $\mathbf{D}$ is full rank because it so clearly illustrates the physical aspects of the control law when the weighting matrices of Eq. (6) are used. First the solution is discussed in spatial variables and then in modal form.

A. Spatial Form of the Energy-Based Control Law

If we restrict our attention to the case where $\mathbf{C} = 0$ (i.e., no structural damping) substituting Eqs. (2) and (6) into Eq. (5) yields

$$\mathbf{P} = \begin{bmatrix} 2\zeta\mathbf{G} & \eta\mathbf{M} \\ \eta\mathbf{M} & \zeta\mathbf{MK}^{-1}\mathbf{G} \end{bmatrix} \tag{7}$$

where

$$\eta = \sqrt{1 + \alpha} - 1$$

$$\zeta = \sqrt{2\eta + \beta}$$

$$\mathbf{G} = \mathbf{M}^{1/2}[\mathbf{M}^{-1/2}\mathbf{K}\mathbf{M}^{-1/2}]^{1/2}\mathbf{M}^{1/2}$$

Equation (7), when substituted into Eq. (4), yields the optimal control law vector to be

$$\mathbf{D}u = -\eta \mathbf{K}q - \zeta \mathbf{G}\dot{q} \tag{8}$$

Theoretically, with **D** a full-rank matrix, the closed-loop eigenvalues and eigenvectors can be placed anywhere. Hence it is interesting to note where the optimal control law places the eigenvalues and eigenvectors when minimization of energy is used as a cost index.

The first term of Eq. (8) yields a force directly proportional to the internal stiffness of the structure. Note that, as α is increased, we increase the speed of response of the structure by essentially multiplying the eigenvalues of the structure by $(1+\eta)$. The second term in Eq. (8) produces equal damping in all modes of the structure. Critical damping for a structure is given by 2**G**; thus, the damping ratio for each mode of the closed-loop system is

$$\xi = \sqrt{(\eta+\beta)/[2(1+\eta)]} \tag{9}$$

The pole locations are found by multiplying each of the open-loop poles ($\pm i\omega_j$) by a scalar, such that the closed-loop poles ($a_j \pm ib_j$) are given by

$$a_j \pm ib_j = [\omega_j\sqrt{1+\eta}]\,[-\xi \pm i\sqrt{1-\xi^2}] \tag{10}$$

Thus choosing weighting matrices of the form given by Eq. (6) moves the poles in a manner that yields equal damping ratios for all modes. The closed-loop eigenvectors are unchanged from the open-loop vectors when **D** is full rank. Rarely will the number of actuators be equal to the number of spatial degrees of freedom; hence, the next section examines a reduced-order modal model.

B. Modal Form of the Energy-Based Control Law

Equation (2) may be transformed from a physical (spatial) basis to a modal basis by the eigenvector matrix **T**, which is orthonormal with respect to **M**. Applying the transformation $q = \mathbf{T}z$ one can derive the first order modal equations to be

$$\dot{\tilde{x}} = \tilde{\mathbf{A}}\tilde{x} + \tilde{\mathbf{B}}u \tag{11}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & \mathbf{I} \\ -\Lambda & -\mathbf{T}^T\mathbf{C}\mathbf{T} \end{bmatrix}, \qquad \tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ \tilde{\mathbf{D}} \end{bmatrix}, \qquad \tilde{x} = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$$

in which Λ is the diagonal eigenvalue matrix and $\tilde{\mathbf{D}} = \mathbf{T}^T\mathbf{D}$. The matrix $\tilde{\mathbf{D}}$ is a square matrix if the number of actuators is equal to the number of modes used in the transformation.

The quadratic functional to be minimized may also be written in modal coordinates as

$$\tilde{\mathbf{J}} = \int_{t_0}^{t_1} \frac{1}{2}(\tilde{x}^T\tilde{\mathbf{Q}}\tilde{x} + u^T\mathbf{R}u)\,\mathrm{d}\tau \tag{12}$$

where

$$\tilde{\mathbf{Q}} = \mathbf{T}^T\mathbf{Q}\mathbf{T} = \begin{bmatrix} \alpha\Lambda & 0 \\ 0 & \beta\mathbf{I} \end{bmatrix}$$

and **R** is written in modal form as $[\tilde{\mathbf{D}}^T\Lambda^{-1}\tilde{\mathbf{D}}]$.

Under the restriction that $\mathbf{C} = 0$ and the number of actuators is equal to the number of modes in the modal model, a closed-form solution for the modal Riccati equation

$$0 = \tilde{\mathbf{Q}} + \tilde{\mathbf{P}}\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T\tilde{\mathbf{P}} - \tilde{\mathbf{P}}\tilde{\mathbf{B}}\mathbf{R}^{-1}\tilde{\mathbf{B}}^T\tilde{\mathbf{P}} \tag{13}$$

is

$$\tilde{\mathbf{P}} = \begin{bmatrix} 2\zeta\Lambda^{1/2} & \eta\mathbf{I} \\ \eta\mathbf{I} & \zeta\Lambda^{-1/2} \end{bmatrix} \tag{14}$$

Equation (14) yields the optimal modal control law vector to be

$$\tilde{\mathbf{D}}u = -\eta\Lambda z - \zeta\Lambda^{1/2}\dot{z}$$

or in spatial coordinates the control law vector is

$$\mathbf{D}u = -\eta\mathbf{D}\tilde{\mathbf{D}}^{-1}\Lambda\mathbf{T}^T\mathbf{M}q - \zeta\mathbf{D}\tilde{\mathbf{D}}^{-1}\Lambda^{1/2}\mathbf{T}^T\mathbf{M}\dot{q} \tag{15}$$

Note that Eq. (15) reduces to Eq. (8) when no modes are truncated (i.e., **T** is a square matrix, $\mathbf{D}\tilde{\mathbf{D}}^{-1} = \mathbf{T}^{-T} = \mathbf{MT}$).

By choosing the weighting matrices to yield consistent physical units in the quadratic functional, the control depends only on two scalar gains and the structural properties. Even for the more general case when the number of actuators is not equal to the number of degrees of freedom in the discrete model (i.e., when no closed-form solution is generally available), the control will still be dependant only on two scalar gains and the structural properties. Such a control law permits more physical insight into the closed-loop system and, as will be discussed in the following sections, is amenable to optimization by structural tailoring.

IV. Structural Tailoring to Optimize Closed-Loop Performance

To minimize the functional of Eq. (3) with respect to some set of design variables, the first step involves evaluation of the quadratic cost function which becomes the minimization objective in the optimization. Since a closed-form solution for the Riccati equation exits, the cost function may be expressed in terms of the initial state, that is, the state when the control system is activated.

$$J = \int_{t_0}^{t_1}\left(\frac{1}{2}x^T\mathbf{Q}x + \dot{u}^T\mathbf{R}u\right)\mathrm{d}\tau = \frac{1}{2}x_0^T\mathbf{P}x_0 \tag{16}$$

Thus, to evaluate the cost function, we must estimate the initial state of the structure. Fortunately, for open-loop structures, we can estimate the initial state assuming the structure is excited by external disturbances. The following sections describe a quasistatic measure of the structural response due to unknown disturbances. This approximate measure of system response is then used to compute the magnitude of the cost function given by Eq. (16).

A. Approximation of Closed-Loop Response due to Unknown Disturbance

To minimize Eq. (16), the peak magnitude of the displacement vector q and velocity vector $\dot{q}$ have been approximated based on the open-loop modal response due to an external step load. Given an undamped structural system subjected to an external force

$$\mathbf{M}\ddot{q} + \mathbf{K}q = f$$

the peak response can be computed by a summation of the linear modes of vibration. For a step load, the peak displacement and velocity can be easily found to be proportional to

$$q \propto \mathbf{T}\Lambda^{-1}\mathbf{T}^Tf, \qquad \dot{q} \propto \mathbf{T}\Lambda^{-1/2}\mathbf{T}^Tf \tag{17}$$

To estimate the effects of f in this study, we expand f into an orthonormal basis of the form

$$f = \mathbf{M}^{1/2}\mathbf{T}\gamma \tag{18}$$

where γ is a vector of modal-force participation coefficients. The modal-force participation coefficients are dependant on the disturbance; however, if no disturbance information is known, equal participation of all modes can be assumed. For the study herein, $\gamma_j = 1$ for all modes.

Substituting Eq. (18) into (17) and writing in state vector form we derive

$$x = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} \propto \begin{Bmatrix} \mathbf{T}\Lambda^{-1}\mathbf{U}\gamma \\ \mathbf{T}\Lambda^{-1/2}\mathbf{U}\gamma \end{Bmatrix} \tag{19}$$

where

$$\mathbf{U} = \mathbf{T}^T\mathbf{M}^{1/2}\mathbf{T} \tag{20}$$

Equation (19), albeit approximate, gives a measure of the peak magnitude of the displacement and velocity response of the structural system. Expressing f in the orthonormal basis of Eq. (18) is approximate to the extent that f can be represented by a truncated set of eigenvectors. Expansion of f in the eigenvector basis makes interpretation of the underlying physics much more transparent.

B. Approximate Cost Function

Equation (16) can now be explicitly stated in terms of structural parameters, two scalar gains and the assumed modal-force coefficients γ. Substituting Eqs. (19) and (14) into (16) gives the following cost function

$$J \propto \left(\eta + \frac{3}{2}\zeta\right)\gamma^T\mathbf{U}^T\Lambda^{-3/2}\mathbf{U}\gamma \tag{21}$$

Thus, the optimal tailoring for the case of an actuator for each controlled mode is that which will minimize $\mathbf{U}^T\Lambda^{-3/2}\mathbf{U}$, which is inversely proportional to the frequency cubed. These results are discussed more fully in the next section with the aid of numerical examples.

V. Results and Discussion

The previous section described an approximate cost function, which minimizes the control energy necessary to achieve vibration suppression. The objective herein is to minimize Eq. (21) by structural tailoring. This section presents results for a clamped-pinned beam and a cantilevered truss beam. Both open- and closed-loop simulations are presented to demonstrate the effect of structural tailoring. The optimization procedure is discussed first and followed by the numerical examples.

A. Structural Tailoring Optimization Procedure

Minimization of Eq. (21) has been performed by tailoring appropriate structural design variables, such as member thickness, to maximize the weighted frequency cubed of selected modes. The contribution of each mode to the objective function is $(U_j\gamma_j/\omega_j^{3/2})^2$. (Note U_j is calculated from Eq. (20) with $\mathbf{T}$ consisting of only the jth eigenvector.) This weighting readily shows that the low-frequency modes contribute most to the cost index. Hence, usually only the low-frequency modes need to be considered in the tailoring process.

Constrained optimization was performed using the ADS[13] system of optimization routines. Eigenvalue sensitivities were computed by finite differences. Since the tailoring procedure is based on the open-loop eigenvalues and eigenvectors, only a finite-element modeling algorithm to compute mass and stiffness matrices, a real eigenvalue algorithm for eigenproblem analysis, and an optimization algorithm for constrained function minimization are required. The following results have also used an implicit integration algorithm and a modal control law synthesis algorithm for transient simulation of the open- and closed-loop systems. All simulations have been computed in spatial coordinates.

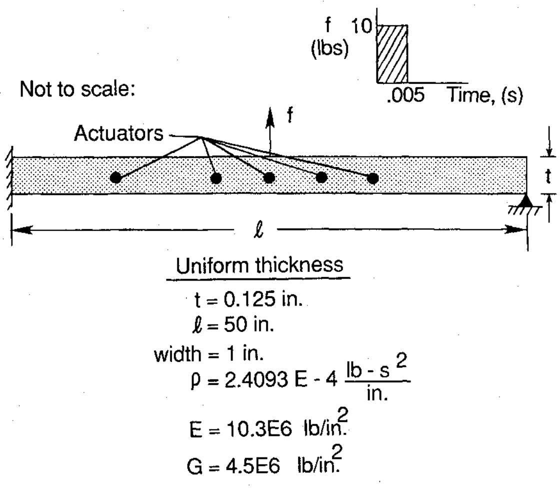

Fig. 1 Uniform clamped-pinned beam.

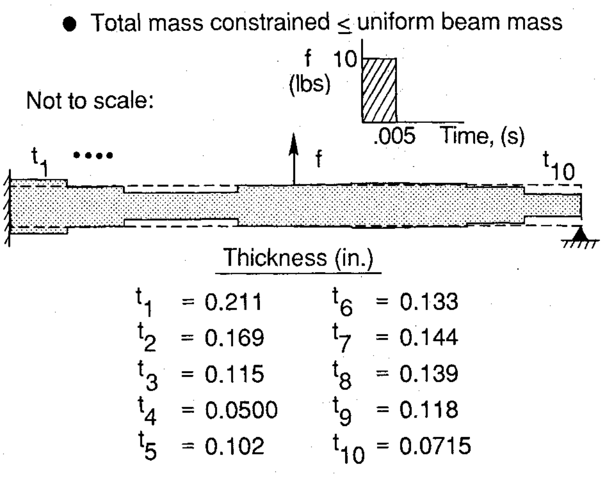

Fig. 3 Tailored clamped-pinned beam.

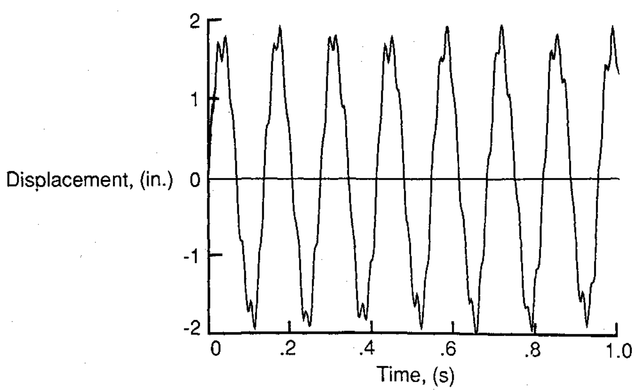

Fig. 2 Open-loop response of uniform beam.

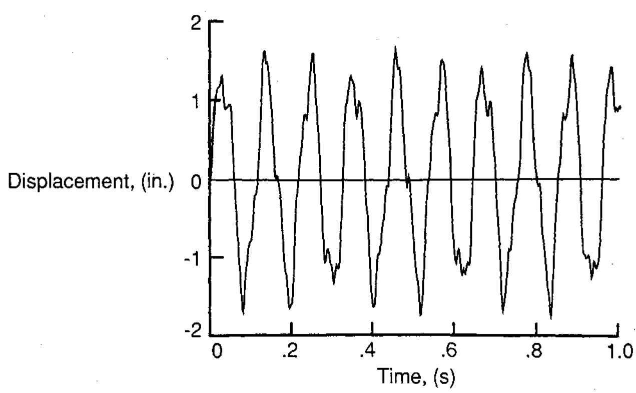

Fig. 4 Open-loop response of tailored beam.

B. Clamped-Pinned Beam Results

The beam shown in Fig. 1 has been studied to determine the effects of structural tailoring on the dynamic performance and feedback control of the beam. Ten Timoshenko beam elements were used in the discrete model. The beam is subjected to a step load at the center of 10 lb for 5 ms. Figure 2 shows the undamped transient response of the uniform beam to this loading condition.

The thickness of each element was tailored to minimize the cost index of Eq. (21). Only the first three modes were considered as the higher modes contributed little to the cost function. An active constraint in the minimization procedure was that the total mass of the tailored beam was to be less than or equal to the mass of the uniform beam. Figure 3 shows the tailored beam and lists the thickness of each element. The undamped transient response of the tailored beam, depicted in Fig. 4, shows a significant reduction in vibration amplitude.

To determine the effect of structural tailoring on the work done by the actuators, the control law given by Eq. (15) was used. Five modes were used to compute the optimal control law for the five actuators located as shown in Fig. 1. For simulation purposes, a perfect estimation of the state has been assumed. The control is turned on at 0.1 s and must reduce the vibration amplitude to less than 0.10 in. within 0.5 s after active control begins. For the examples herein, only rate feedback was used, i.e., $\alpha = 0$.

Figure 5 shows the closed-loop transient response of the uniform and tailored beams. Both beams satisfy the preceding performance requirement. Figure 6 shows a reduction of 37% in total control work for the tailored beam. Thus, the work of the actuators can be significantly reduced for some actively controlled structures by simply tailoring the structure to minimize a function inversely proportional to the vibration frequency cubed. Although such tailoring may be advantageous to other control laws, the physical based control law presented herein maximizes the effect of tailoring to reduce control work.

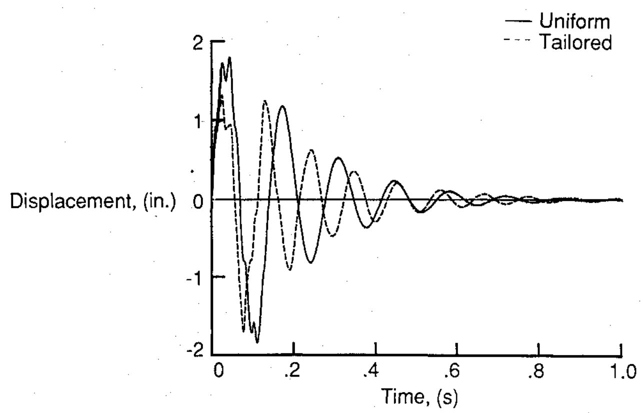

Fig. 5 Closed-loop response of uniform and tailored beams.

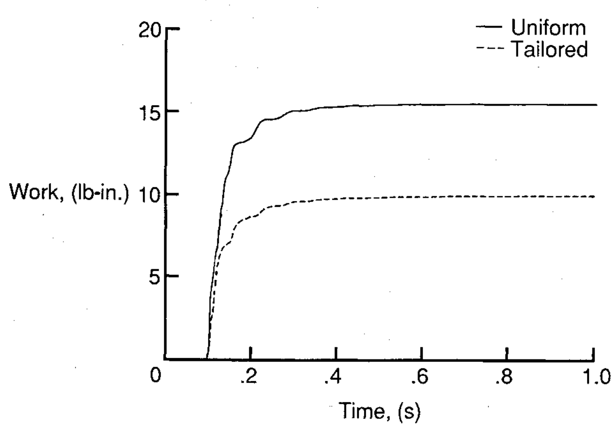

Fig. 6 Controller work of uniform and tailored beams.

C. Cantilever Truss-Beam Results

The truss beam shown in Fig. 7 has been used to demonstrate that even large-order systems can be easily tailored to reduce control cost using the method proposed in this study. The three longeron, single-laced truss was modeled by finite elements with one Timoshenko beam element from joint-to-joint. The model had 165 nodes and 990 degrees of freedom. All members are tubular with the inside diameter set equal to 75% of the outside diameter. Three design variables, the outside diameters of the batten, and diagonal and longeron elements are used to tailor the structure. In addition to the mass constraint mentioned previously, the first pinned-pinned frequency of each element was constrained to be more than 100 times the first global bending frequency.

Table 1 lists the truss-beam properties as well as the nominal and tailored tube diameters. The nominal outside tube diameters were chosen such that the first pinned-pinned beam frequency of each element was 100 Hz. The beams are subjected to a step load at two points as shown in Fig. 7 of 10 lb magnitude for a duration of 100 ms. Figure 8 shows the undamped response of the uniform and tailored beam. Again, the tailored beam shows a substantial decrease in vibration amplitude.

A seven-mode control law was used to study the control cost for each truss beam necessary to reduce the vibration amplitude to 0.025 in. within 10 s after active control began. Six

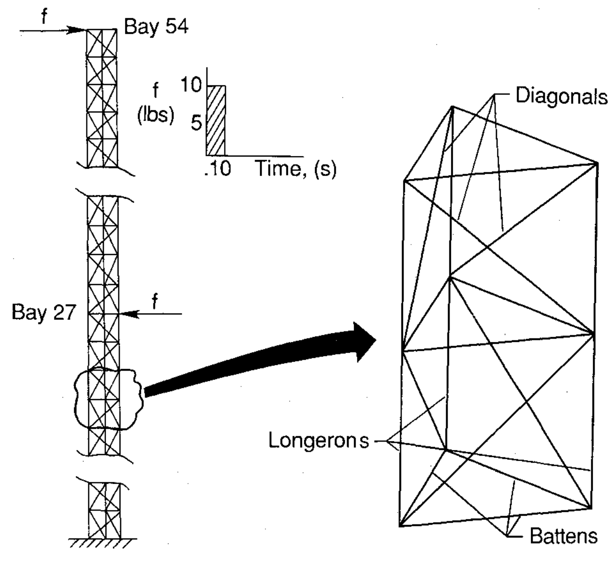

Fig. 7 Cantilevered truss beam.

Table 1 Truss-beam properties

Outside tube diameter (in.)	Nominal	Tailored
longeron	0.789	1.717
diagonal	1.707	1.284
batten	0.918	0.640

Material properties

$\rho = 1.5285\,e - 4$ lb $-$ s^2/in.^{4}
$E = 40.0\,e + 6$ lb/in.^{2}
$G = 2.4\,e + 6$ lb/in.^{2}

Element lengths

longeron = 44.25 in.
diagonal = 65.09 in.
batten = 47.73 in.

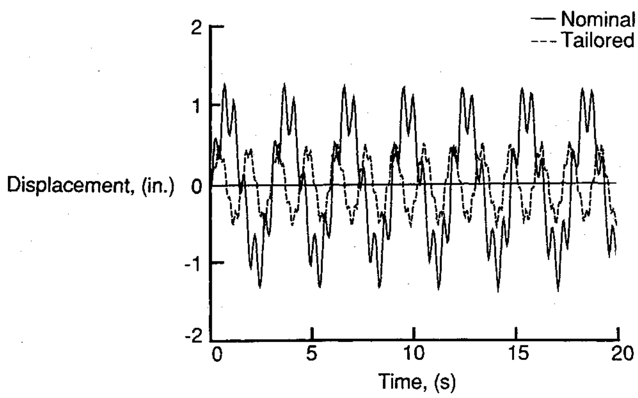

Fig. 8 Open-loop response of nominal and tailored truss beams.

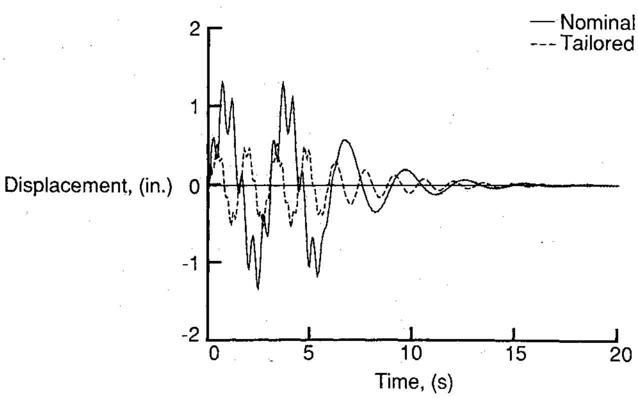

Fig. 9 Closed-loop response of nominal and tailored truss beams.

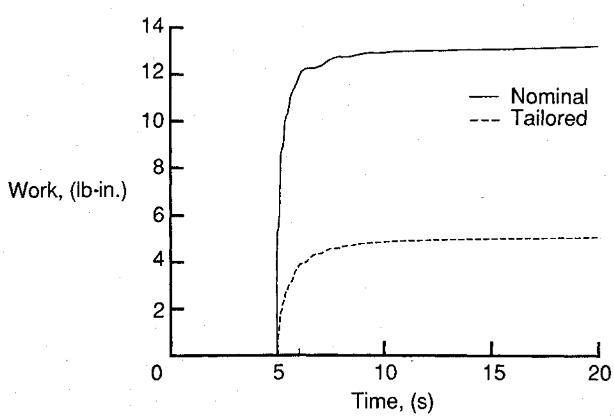

Fig. 10 Controller work of nominal and tailored truss beams.

actuators were located in both bending planes at bays 27, 53, and 54. One additional actuator was located in the plane of the external force at bay 28. Active control was initiated 5 s after the load was applied. Figure 9 shows that both truss beams meet the performance requirement; however, the tailored beam requires 56% less total actuator work as shown in Fig. 10. Thus, structural tailoring can produce substantial reductions in controller energy. Moreover, the structural procedure presented in this study is simple enough to be applied to large systems.

VI. Summary

Structural tailoring of actively controlled structures has significant advantages over pure feedback control. However, considerable effort is required to incorporate physical insight as part of the synthesis process in order to take full advantage of combined structural tailoring and active feedback control. Results herein have shown that a set of physically based weighting matrices in the LQR performance criterion leads to a control law that has direct physical meaning. This physical understanding of the control law leads to the development of the proper structural tailoring objective. For minimization of controller energy, the tailoring objective is inversely proportional to the open-loop vibration frequencies cubed. This objective is simple enough to be applied early in the structure/control design process.

Numerical results for a beam subjected to an external disturbance have been presented. Structural tailoring of the beam thickness reduced the control energy by 37% from that of the uniform beam. In addition, numerical results for a more realistic space truss beam were presented. Structural tailoring of the truss beam reduced the actuator work by 56%.

Future studies are needed to address more general control laws where the number of actuators differs from the number of model degrees of freedom. Such control laws will result even for modal-space control when actuator dynamics are considered. In this case, the control laws will produce closed-loop eigenvectors that differ from the open-loop eigenvectors.

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